Forecast requirements for house temperature control with flexible energy prices

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ENFOR A/S
Lyngsø Allé 3
DK-2970 Hørsholm
www.enfor.dk

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Contents

Summary

The present report was prepared as part of the FlexPower project[1]. Due to the increasing penetration of wind energy the aim of the project is to examine the use of flexible electricity prices to increase the controllability of the electric system and hence balance out the loads. The present report is related to work package 5 dealing with forecast requirements for control. A house temperature control setup is considered where the controller should optimally trade-off comfort and economical cost. In order to do this forecasts of electricity prices and external conditions (e.g. out-door temperature) are highly valuable. In this report the requirements for the forecasts are clarified when using an optimal control methodology.

Nomenclature

The following notation is used frequently in the report:

Symbol	Description
$\mathbb{E}\{X\}$	Expectation of random variable X
$\mathbb{V}\{X\}$	Variance of random variable X
$\mathbb{C}\{X,Y\}$	Covariance of random variables X and Y
$\mathbb{Q}^p\{X\}$	$(p \cdot 100\%)$-quantile of random variable X
$\mathbb{P}\{X \in \mathcal{A}\}$	probability of random variable X being in $\mathcal{A}$
$\mathcal{Y}$	set of information

If a statistics (such as $\mathbb{E}$) of a random variable X is conditional to a set of information $\mathcal{Y}$, this is denoted by $\mathbb{E}\{X|\mathcal{Y}\}$. With words, this means that in giving the expectation of X all the information in $\mathcal{Y}$ has been accounted for.

1 Introduction

Due to the stochastic nature of wind energy the increasing penetration of wind energy sets a high demand for balancing the loads in the electrical grid. Today the regulating power to balance the system comes from the central power plants and import/export to hydro power plants in Norway and Sweden. However, the capacity of central power plants is likely to decrease with the increase of wind energy. One approach to gain regulating power is to extend the energy market such that that the electricity demand can be regulated. In the FlexPower project it

[1] Energinet.dk/PSO project “FlexPower”, nr. 2010-1-10486

is proposed to use a 5 min. electricity price to give an incentive for adjusting the electricity consumption and small scale electricity generation.

In this report a case study concerning temperature control is analysed. The basic idea is to optimally trade-off comfort (in terms of temperature) and the associated economical cost. Given that the house has a large capacity or has a build in storage, the aggregation of a large number of houses may lead to significant regulating power.

The 5 min. electricity price will naturally be governed by several factors, however, to a large extend the evolution of the price will be stochastic. In order that the controller can make optimal decisions it is essential that a forecast of the electricity price is available. This report investigates the requirements for the electricity price forecast - i.e. should the expected evolution of the price be forecasted or should information about the forecast uncertainty, etc. be available.

The temperature of a house is naturally influenced by the external conditions (outdoor temperature, solar radiation, wind speed, etc.). Again, the controller will benefit by having knowledge of the future of the external conditions. Forecast requirements for the external conditions are therefore also investigated.

It is not the scope of this report to fully develop the actual control scheme, although, in order to provide conclusions regarding forecast requirements, some assumptions need to be made. The general consensus in the project is towards an optimal control scheme which trade-off economical cost and discomfort. A natural approach is therefore to consider the so-called rolling horizon methodology[2, 3]. Rolling horizon control (also denoted moving horizon or receding horizon control) is a model based control method in which the optimisation horizon is fixed in length and shifted forward as time progresses. At every time-step the optimal control trajectory is calculated and the first control move actuated. Since, both the electricity price and the external conditions are assumed to pertain to stochastic processes, the control problem is considered in its stochastic formulation. Based on a selection of stochastic rolling horizon formulations it is investigated what information to forecast to optimally account for future prices and external conditions. See [1] for a general reference on stochastic control.

The report is organised as follows: In Sec. 2 a simple mathematical description of the house (FlexHouse) is introduced. Furthermore, a short description of the control problem and forecasts is given. In Sec. 3 the forecast requirements are investigated using different stochastic variants of the control problem. The final conclusions and perspectives are given in Sec. 4 and 5 respectively.

2 System, control and forecasts

In this section a simple model of a house system is introduced which is the basis for the analysis. The control problem and the inherent optimisation problem is sketched. Finally, the forecasts that will be available for the controller are described.

2.1 Simple house model

A simple system is considered consisting of a house with a radiator. The radiator power Q can be controlled such that the indoor temperature T can be kept close to a desired level T^d. It is assumed that the temperature dynamics of the house pertains to a linear system with inputs being radiator power Q and outdoor temperature T^o. The output is simply the indoor temperature.

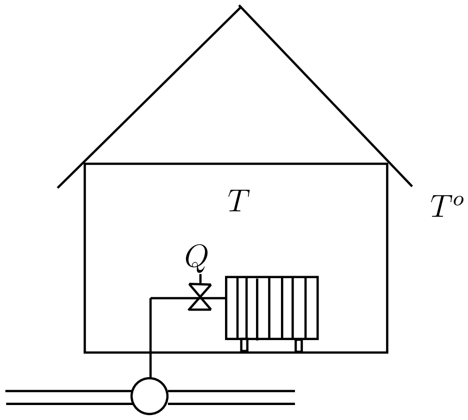

Figure 1: Sketch of system. The temperature of the house depends on the radiator consumption Q and the outdoor temperature T^o.

Under stationary conditions the power consumption of the house will be

$$Q = U \cdot A \cdot (T - T^o)$$

where U is the overall heat transfer coefficient of the house and A is the area of the house boundaries (walls, etc.) It is assumed that the response of the indoor temperature to a change in radiator power pertains to a first order transfer function with an appropriate time constant ($\tau_Q \sim 1 - 2$ hours):

$$H(s) = (UA)^{-1} \frac{1}{\tau_Q s + 1}$$

Equivalently, the response to changes in outdoor temperature is modelled as a first order transfer function with an appropriate time constant ($\tau_o \sim 12 - 15$ hours):

$$H_o(s) = \frac{1}{\tau_o s + 1}$$

The total dynamics is given by H and H_o in parallel

$$T = H(s)Q + H_o(s)T^o. \tag{1}$$

By straight forward manipulations the system can be put on the following state-space form:

$$\dot{T}^{iQ} = -\frac{1}{\tau_Q}T^{iQ} + (UA)^{-1}\frac{1}{\tau_Q}Q \tag{2}$$

$$\dot{T}^{io} = -\frac{1}{\tau_o}T^{io} + \frac{1}{\tau_o}T^o \tag{3}$$

$$T = T^{iQ} + T^{io} \tag{4}$$

where T^{iQ} and T^{io} are the states of the system representing the temperature contribution from the radiator and outdoor temperature respectively. The temperature deviation from desired is naturally

$$\Delta T = T^d - T \tag{5}$$

2.2 Control problem

It is assumed that a controller will be designed to minimise a trade-off between comfort (i.e. temperature deviations) and the price associated with control actions. Roughly

$$\boxed{\text{cost(actual temp. - desired temp.)}} + \boxed{\text{power price} \times \text{power consump.}}$$

For future reference, the first term will be denoted the *discomfort cost* and the second term the *economical cost*. This control problem can be formulated as a finite horizon optimisation problem will a rolling horizon[3]. The term *rolling* refers to the fact that the optimisation problem is solved at every time step with a fixed horizon length (see Fig. 2). Since, for practical reasons, the horizon is not infinite it is necessary to re-calculate the optimal control trajectory whenever time has progressed. Furthermore, conditions may have changed due to disturbances and recalculating the control at every sample time hence introduces feedback. Only the first control move of the optimal input trajectories is actuated.

In the following it is assumed that a quadratic penalty is associated with discomfort (temperature deviations). The penalty associated with the use of radiator power Q is simply the

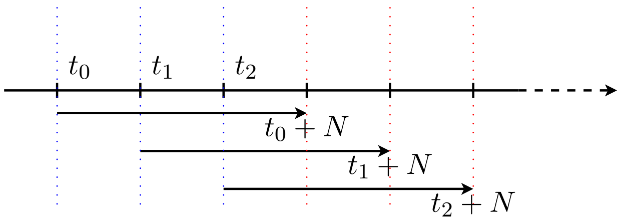

Figure 2: Illustration of the rolling horizon methodology. At every time step the same fixed length optimisation problem is solved.

economical cost $P \cdot Q$. The optimisation problem hence take the following form in discrete time:

$$J_t^* = \min_{Q_t, Q_{t+1}, \ldots, Q_{t+N}} \left\{ J_t = \sum_{k=t}^{t+N} \left(P_k Q_k + r \Delta T_k^2 \right) = J_t^{\text{eco}} + r J_t^{\text{discomf}} \right\} \tag{6}$$

subject to the system dynamics and other constraints such as bounds on the control power Q. The cost term J_t^{eco} and J_t^{discomf} naturally refers to the economical and discomfort cost respectively. The parameter r is a tuning parameter for getting the desired trade-off between economy and comfort. Assuming that the unit of r is $\frac{kr}{{}^\circ C^2}$ the total cost will be given in terms of money.

In order to achieve the above objective the controller will as a minimum need observations of indoor temperature and the power price. The power price P and outdoor temperature T^o are in some sense random variables. To achieve the best possible control it is therefore advantageous to have information about the future evolution of these variables i.e. forecasts.

Note: How temperature deviations affects the discomfort is naturally difficult to quantify without doing actual experiments on people. A good starting point is however to simply consider penalising quadratic deviations from the desired temperature - this is also very convenient from a mathematical perspective.

2.3 Forecasts

In the following it is assumed that the power price is forecasted with a 5 min. resolution as this is integral to the objectives of the FlexPower project. It is also assumed that the *effect* of the outdoor temperature is forecasted *but* not necessarily the outdoor temperature itself.

Based on the physical equations of the system it is natural to require that a forecast of the outdoor temperature is available in order to anticipate changes in temperature. The system model is however highly simplified and the temperature T^o should be seen as representing the *effective* temperature that affects the house. This temperature hence represents the combined effect of the temperature, wind and solar radiation (and possibly more factors). Forecasts of these variables can be obtained from MET offices, but will likely be biased since they do not reflect the local conditions. Two houses which lie in the same grid cell of the MET forecast could be placed in quite different environments - e.g. on a hill or besides a lake. In order to calibrate for the local conditions it is necessary to have actual observations of all these variables. This adds to the complexity of the technical installation.

Another approach is to forecast what in the following will be called the consumption demand Q^d. The consumption demand is the power that must be dissipated in the house to compensate for the climate to maintain the desired temperature level. Temperature deviations from desired are hence given directly as

$$\Delta T = (UA)^{-1} Q^d - T^{iQ} \tag{7}$$

where T^{iQ} is the temperature contribution from the radiator (see eq. (2)-(4)). Hence, the desired temperature is maintained if the radiator manages to dissipate the power Q^d into the house. In order to forecast the consumption demand Q^d it is required to have observations of the radiator consumption Q and the indoor temperature - information which is already available from the controller. The availability of MET forecasts is naturally also a requirement to maintain high quality forecast.

It is quite easy to integrate the comsumption demand with the system description (2)-(4). In fact simple manipulations show that Q^d and T^{io} are related as follows:

$$T^{io} = T^d - (UA)^{-1} Q^d \tag{8}$$

Hence, if the forecast model targets the comsumption demand Q^d it is not necessary for the controller to model the dynamics relating the outdoor temperature to the indoor temperature (eq. (3)) as this is captured by the forecasting model. The system description hence becomes:

$$\dot{T}^{iQ} = -\frac{1}{\tau_Q} T^{iQ} + (UA)^{-1} \frac{1}{\tau_Q} Q \tag{9}$$

$$T^{io} = T^d - (UA)^{-1} Q^d \tag{10}$$

$$T = T^{iQ} + T^{io} \tag{11}$$

The consumption demand Q^d represents an external signal/disturbance in equations (9)-(11), as do the outdoor temperature T^o in equations (2)-(4). The derivation of the forecast requirements concerning Q^d and T^o will therefore essentially be equivalent. As shown in Appendix A the associated system descriptions can be unified on the following form (in discrete time for convinience):

$$\mathbf{x}_{t+1} = \mathbf{A}\mathbf{x}_t + \mathbf{B}Q_t + \mathbf{G}d_t \tag{12}$$

$$\Delta T_t = \mathbf{C}\mathbf{x}_t + \mathbf{D}T_t^d + \mathbf{H}d_d \tag{13}$$

where $\mathbf{x}$ is the state of the system and d represents either the outdoor temperature T^o or the consumption Q^d - in general the *external conditions*. To proceed in a general manner the system description (12)-(13) is used in the derivation of the forecast requirements.

3 Forecast requirements

A forecast is not just a forecast. Since the future is uncertain a *statistical* forecast communicates a statistics of the variables under consideration e.g. the expected value. The uncertainty of the future can likewise be communicated by forecasting the standard deviation as illustrated in Fig. 3. Since the external conditions represented by d and the price P both have a suitable

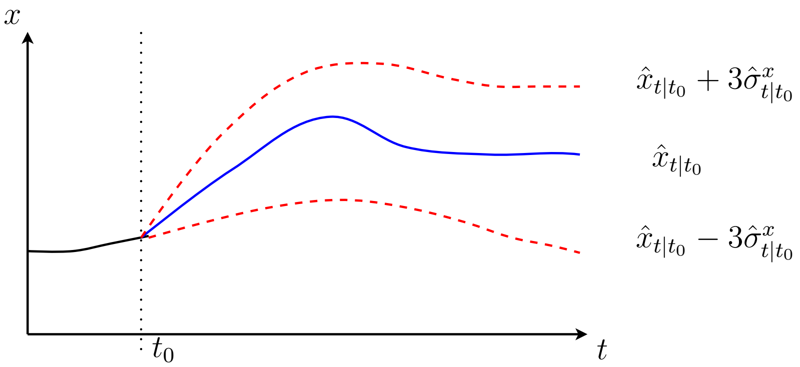

Figure 3: Forecast of expected value $\hat{x}$ and uncertainty $\hat{\sigma}^x$ at time t_0

interpretation as random variables it is relevant to clarify what statistic of these random variables is the optimal information for the control problem. To answer this question it is necessary to interpret the optimisation problem as a stochastic problem.

The cost function in equation 6 is itself a random variable since it depends on d and P. The natural formulation of the problem is therefore as a stochastic optimisation problem. The problem can essentially not be solved unless a statistics of the cost is evaluated. The first question should therefore be: what statistics of the cost should be evaluated? In the stochastic control literature the standard statistics is simply the expected value. There are several reasons for this. First of all it is a meaningful statistics in most circumstances. Secondly, for linear systems and under certain assumptions on disturbances the solution is equivalent to the deterministic problem. The conditional expectation of the cost is

$$\mathbb{E}\{J_t|\mathcal{Y}_t\} = \mathbb{E}\{J_t^{\text{eco}}|\mathcal{Y}_t\} + r\mathbb{E}\{J_t^{\text{discomf}}|\mathcal{Y}_t\}$$

where $\mathbb{E}\{\cdot|\mathcal{Y}_t\}$ denotes the expectation conditional to all information available at time t. Since, the problem is stochastic it might also be of interest to minimise the variation of the economical price and consequently append $\mathbb{V}\{J_t^{\text{eco}}|\mathcal{Y}_t\}$ to the cost:

$$\mathbb{E}\{J_t^{\text{eco}}|\mathcal{Y}_t\} + r\mathbb{E}\{J_t^{\text{discomf}}|\mathcal{Y}_t\} + r_v\mathbb{V}\{J_t^{\text{eco}}|\mathcal{Y}_t\}$$

The parameter r_v is a tuning parameter. In the following it is clarified what forecasts requirements are associated with each of the cost terms when minimising the total cost. It is well known that minimising a function it is necessary to look at the gradients with respect to the free variables (which in the unconstrained case should be 0). A forecast will therefore in the following be classified as required if it appears in the gradients of the cost function. Since the gradient of the sum is equal to the sum of the gradient, the terms $\mathbb{E}\{J_t^{\text{eco}}|\mathcal{Y}_t\}$, $\mathbb{E}\{J_t^{\text{discomf}}|\mathcal{Y}_t\}$ and $\mathbb{V}\{J_t^{\text{eco}}|\mathcal{Y}_t\}$ are considered individually in the following.

Besides the stochastic cost it is also examined what the requirements will be if the immediate cost is constrained in the optimisation problem i.e. a constraint on the form $P \cdot Q \leq b$. Since the price is a random variable this will be incorporated as a probabilistic constraint. More specifically it is required that the immediate cost is less than b with a certain probability p throughout the forecast horizon:

$$\mathbb{P}\{P_{t+h}Q_{t+h} \leq b|\mathcal{Y}_t\} = p \ , \quad h = 0, 1, \ldots, N$$

3.1 Forecast requirements associated with expected discomfort

The conditional expectation of the discomfort cost can be split into two parts:

$$\mathbb{E}\{J_t^{\text{discomf}}|\mathcal{Y}_t\} = \sum_{k=t}^{t+N} \mathbb{E}\{\Delta T_k^2|\mathcal{Y}_t\} = \sum_{k=t}^{t+N} \mathbb{E}\{\Delta T_k|\mathcal{Y}_t\}^2 + \sum_{k=t}^{t+N} \mathbb{V}\{\Delta T_k|\mathcal{Y}_t\}$$

The cost can be expanded even further by utilising the equations for the dynamic system (12)-(13) giving the following result:

$$\sum_{k=t}^{t+H} \mathbb{E}\{\Delta T_k|\mathcal{Y}_t\}^2 = \sum_{k=t}^{t+H} \left[\mathbf{C}\mathbf{A}^k \mathbb{E}\{\mathbf{x}_t|\mathcal{Y}_t\} + \mathbf{C} \sum_{i=0}^{k-1} \mathbf{A}^{k-1-i} \left(\mathbf{B}Q_{t+i} + \mathbf{G}\mathbb{E}\{d_{t+i}|\mathcal{Y}_t\} \right) + \mathbf{D}T_k^d \right. \\ \left. + \mathbf{H}\mathbb{E}\{d_k|\mathcal{Y}_t\} \right]^2 \tag{14}$$

$$\sum_{k=t}^{t+H} \mathbb{V}\{\Delta T_k|\mathcal{Y}_t\} = \sum_{k=t}^{t+H} \mathbb{V} \left\{ \mathbf{C}\mathbf{A}^k \mathbf{x}_t + \mathbf{C} \sum_{i=0}^{k-1} \mathbf{A}^{k-1-i} \mathbf{G} d_{t+i} + \mathbf{H} d_k \middle| \mathcal{Y}_t \right\} \tag{15}$$

The second term in the cost (equation (15)) does not have a dependency on the control variable and will therefore not have any influence on the forecast requirements. It is however quite easily verified that the gradient of the first term (equation (14)) depends on the conditional expectation of the external conditions d. The forecast requirement is therefore:

$$\boxed{\text{Required forecast:} \qquad \mathbb{E}\left\{d_{t+k}|\mathcal{Y}_t\right\}, \ k = 0, 1, \ldots, N}$$

The gradient will actually also depend on the conditional expectation of the state $\mathbb{E}\{\mathbf{x}_t|\mathcal{Y}_t\}$ at time t. This is usually a quantity which is estimated internally in the controller.

3.2 Forecast requirements associated with expected economical cost

The economical cost is independent of the system dynamics. The required forecast statistics of the price P is hence given directly by evaluating the expected economical cost:

$$\begin{aligned}\mathbb{E}\{J_t^{\text{eco}}|\mathcal{Y}_t\} &= \mathbb{E}\left\{\sum_{k=t}^{t+N}(P_k Q_k)\middle|\mathcal{Y}_t\right\} \\ &= \sum_{k=t}^{t+N}(\mathbb{E}\{P_k|\mathcal{Y}_t\}Q_k)\end{aligned}$$

It is seen that the expected economical cost can be evaluated if the conditional expectation of the price is known. The gradient of the cost clearly depends on the expected price consequently giving the following requirement.

$$\boxed{\text{Required forecast:} \qquad \mathbb{E}\{P_{t+k}|\mathcal{Y}_t\},\ k=0,1,\ldots,N}$$

3.3 Forecast requirement associated with variance of economical cost

The conditional variance of the economical cost can be expanded as follows:

$$\mathbb{V}\{J_t^{\text{eco}}|\mathcal{Y}_t\} = \mathbb{V}\sum_{k=t}^{t+N}(P_k Q_k)|\mathcal{Y}_t\} \tag{16}$$

$$= \sum_{k=t}^{t+N} Q_k^2\mathbb{V}\{P_k|\mathcal{Y}_t\} + 2\sum_{i=t}^{t+N-1}\sum_{j=i+1}^{t+N}(Q_iQ_j\mathbb{C}\{P_i,P_j|\mathcal{Y}_t\}) \tag{17}$$

Since, in $\mathbb{V}\{a+b\} = \mathbb{V}\{a\}+\mathbb{V}\{b\}+2\mathbb{C}\{a,b\}$ the forecast requirement is more intricate than for the expectation. Evaluating the gradient of equation (17) with respect to the control actions it is easily verified that the gradient depends on conditional variance-covariance of prices at different forecast times. Hence the forecast requirement is:

$$\boxed{\text{Required forecast:} \qquad \mathbb{C}\{P_{t+k},P_{t+j}|\mathcal{Y}_t\},\ k=0,1,\ldots,N}$$

If the variance of the economical cost was included as a *constraint* rather than a cost the forecast requirements would be the same.

A note on relevance: It can be argued that the variance of the cost is of little value in the case study: The house-owner evaluates the control performance through the quarterly (or similar) electricity bills. This is a rather long averaging time compared to the horizon length of the controller which might be around 12-24 hours. Hence, the risk (variance) predicted by the controller is not the actual risk of the house owner. Furthermore, it is important to stress that the control action is re-calculated at every time step taking into account the newest information. All in all, the risk accounted for by the controller does not co-inside with, and is likely higher than, the actual risk. In the nature of keeping things simple it is arguably most reasonable only to consider expectation as the cost statistics.

3.4 Forecast requirement associated with constraint on the immediate economical cost

As previously stated a constraint on the immediate cost is adequately formulated as a probabilistic constraint for each forecast horizon individually:

$$\mathbb{P}\{P_{t+k}Q_{t+k} \leq b|\mathcal{Y}_t\} = p \ , \quad k = 1, 2, \ldots, N$$

Given that the constraint is satisfied, the probability of violating the constraint is $1-p$. Using the definition of a quantile the probabilistic constraint is given as:

$$\mathbb{Q}^p\{P_{t+k}|\mathcal{Y}_t\}Q_{t+k} \leq b \ , \quad k = 1, 2, \ldots, N$$

where $\mathbb{Q}^p\{P_{t+k}|\mathcal{Y}_t\}$ denotes the $(p \cdot 100)$%-quantile of P_{t+k} conditional to information at time t, i.e.:

$$\mathbb{Q}^p\{P_{t+k}|\mathcal{Y}_t\} = \{x \text{ s.t. } \mathbb{P}\{P_{t+k} \leq x|\mathcal{Y}_t\} = p\}$$

Since the constraint depends on the forecasted quantile the forecast requirement is naturally:

$$\boxed{\text{Required forecast:} \qquad \mathbb{Q}^p\left\{P_{t+k}|\mathcal{Y}_t\right\}, \ k = 1, 2, \ldots, N}$$

A note on relevance: Constraining the immediate cost does not have a clear importance in connection to the FlexPower project. Arguably the only reason to include the constraint is not to worry the user with high peaks in the immediate economical cost. One can argue that the immediate cost can be fully controlled since the price is known at the time of control, therefore removing the need for a probabilistic constraint all together. This is, however, not entirely correct; in order to make the optimal decision the controller should take into account that the immediate cost is to be bounded throughout the forecast horizon - and for the future the price is not known. Although of little importance in the FlexPower project, it provides an insight for the situations where the quantile is the required statistics to be communicated by the forecast.

4 Conclusion

The conclusions for the required forecasts are summarised in Table 1

	Cost/constraint	Forecast requirement	Relevance
Discomfort	$\mathbb{E}\{J_t^{\text{discomf}}\|\mathcal{Y}_t\}$	$\mathbb{E}\{d_{t+k}\|\mathcal{Y}_t\}$	**high**
Economy	$\mathbb{E}\{J_t^{\text{eco}}\|\mathcal{Y}_t\}$	$\mathbb{E}\{P_{t+k}\|\mathcal{Y}_t\}$	**high**
	$\mathbb{V}\{J_t^{\text{eco}}\|\mathcal{Y}_t\}$	$\mathbb{C}\{P_{t+k}, P_{t+j}\|\mathcal{Y}_t\}$	little
	$\mathbb{P}\{P_{t+k} \cdot Q_{t+k} \leq b\|\mathcal{Y}_t\}$	$\mathbb{Q}^p\{P_{t+k}\|\mathcal{Y}_t\}$	very little

Table 1: Table of forecast requirements

The statistics used for evaluating the cost/constraint is seen to govern the statistics to be forecasted of the electricity price and external conditions (such as outdoor temperature). In short the conclusions are the following

- Minimisation of the expected discomfort requires forecasts of the expected external conditions (such as outdoor temperature)
- Minimisation of the expected economical cost requires forecasts of the expected price.
- Minimisation of the variance of the economical cost requires forecasts of co-variation of prices for different forecast times.
- Probabilistic restrictions on the immediate cost requires price quantile forecasts.

It is argued in the report that the minimisation of expected discomfort and economical cost are the most relevant for the case study. This is because the controller aim at minimising the total cost as experienced over longer periods (several months). To accomplish this the controller should minimise the conditional expected cost over a rolling horizon long enough to ensure that the immediate decision is not sub-optimal with respect to future (expected) decisions. It might be argued that the controller should also restrict the economical risk associated with the decisions. The relevant economical risk is associated with longer periods as mentioned above. Due to cancelling effects and due to the fact that the price, for which the consumption is actually implemented, is known, penalizing the conditional variances will likely only have minor influence on the relevant economical risk. In any case, expected cost leads to the most simple requirements and is the standard approach in stochastic control.

In the simplified physical model of the house the outdoor temperature represents the influence of the climate. In actual applications the physical models will presumably make use of more meteorological variables such as wind speed and solar radiation. It is argued in the report that using meteorological forecasts directly of these variables might be suboptimal for the applications of interest. The meteorological models work on grid sizes covering areas of several square kilometres and it is likely that within such areas systematic variations due to local effects will exist. For this reason it might be required to calibrate the meteorological forecasts using actual measurements on site climate measurements. Another approach motivated in the report is to forecast the consumption demand which is the required power that must be dissipated in the house to keep the desired temperature. In this case the house itself acts as a sensor and additional sensors for measuring the climate are avoided.

5 Perspectives

The quadratic discomfort cost, although mathematically convenient, is not necessarily a good representation of the actual cost of discomfort. It is likely that there is less discomfort associated with a deviation of $+3$ degrees than a deviation of -3 degrees. Some asymmetry in the cost would therefore be desirable. Asymmetry can indirectly be attained by adding to the problem soft constraints on the expected temperature deviations. This will not affect the forecast requirements. The direct approach would be to replace the quadratic cost with an asymmetric function. A particular simple example is a piece-wise linear cost on temperature deviations. Replacing the quadratic cost will in general lead to other forecast requirements for the external conditions d.

A Derivation of unifying state-space description

In Sec. 2.3 it is proposed to forecast the consumption demand rather than the outdoor temperature. As stated the system equations will be different, however, the general structure remains the same and is given in equations (12)-(13). In this section it is shown how the system descriptions can be put on the general form. In the following the system equations are discretized by approximating the the differentiation w.r.t. time with the difference:

$$\dot{x} \approx \frac{x_{t+t_s} - x_t}{t_s} \tag{18}$$

Throughout the remaining part of the section t will denote normalised sample time rather than absolute time.

If it is desired to forecast the outdoor temperature, the associated continuous time state space description is (2)-(4). Using the approximation (18) the discretized system is trivially put on the form:

$$\begin{bmatrix} T^{iQ}_{t+1} \\ T^{io}_{t+1} \end{bmatrix} = \underbrace{\begin{bmatrix} 1 - \frac{t_s}{\tau_Q} & 0 \\ 0 & 1 - \frac{t_s}{\tau_o} \end{bmatrix}}_{\mathbf{A}} \begin{bmatrix} T^{iQ}_t \\ T^{io}_t \end{bmatrix} + \underbrace{\begin{bmatrix} (UA)^{-1} \frac{t_s}{\tau_Q} \\ 0 \end{bmatrix}}_{\mathbf{B}} Q_t + \underbrace{\begin{bmatrix} 0 \\ \frac{t_s}{\tau_o} \end{bmatrix}}_{\mathbf{G}} T^o_t$$

$$\Delta T_t = \underbrace{\begin{bmatrix} -1 & -1 \end{bmatrix}}_{\mathbf{C}} \begin{bmatrix} T^{iQ}_t \\ T^{io}_t \end{bmatrix} + \underbrace{\begin{bmatrix} 1 \end{bmatrix}}_{\mathbf{D}} T^d_t + \underbrace{\begin{bmatrix} 0 \end{bmatrix}}_{\mathbf{H}} Q^d$$

The state of the system is $\mathbf{x}_t = [T^{iQ}_t \;\; T^{io}_t]^T$ and the disturbance is naturally $d_t = T^o_t$.

If it is desired to forecast the consumption demand, the associated continuous time state space description is (9)-(11). Using the approximation (18) the system is trivially given by:

$$T^{iQ}_{t+1} = \underbrace{\begin{bmatrix} 1 - \frac{t_s}{\tau_Q} \end{bmatrix}}_{\mathbf{A}} T^{iQ}_t + \underbrace{\begin{bmatrix} (UA)^{-1} \frac{t_s}{\tau_Q} \end{bmatrix}}_{\mathbf{B}} Q_t + \underbrace{\begin{bmatrix} 0 \end{bmatrix}}_{\mathbf{G}} Q^d$$

$$\Delta T_t = \underbrace{\begin{bmatrix} -1 \end{bmatrix}}_{\mathbf{C}} T^{iQ} + \underbrace{\begin{bmatrix} 0 \end{bmatrix}}_{\mathbf{D}} T^d + \underbrace{\begin{bmatrix} (UA)^{-1} \end{bmatrix}}_{\mathbf{H}} Q^d$$

The state of the system is $\mathbf{x}_t = T^{iQ}_t$ and the disturbance is naturally $d_t = Q^d_t$.

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