Stochastic control principles

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Agenda

- The house problem revisited
  - Issues with receding horizon solution
- The electric car problem
  - Optimal policy for simplified (dice) example
- Conclusions

House optimization revisited

- Receding horizon methodology
- With a quadratic penalty on temperature deviation + the natural price penalty the following forecasts are required
  - Conditional expectation of consumer consumption (or the outdoor temperature)
  - Conditional expectation of the price
- At any given time *t* the optimal open-loop trajectory is calculated – feedback is indirect by recalculating the trajectory at every sample time.

House optimization con´t

- This approach is not globally optimal in a math. sence – but could very well be the most practical solution
  - MPC does not explicitly go for a control policy – feedback is not explicitly accounted for
  - Not an issue for linear systems with additive gaussian uncertainty – However, the electricity price is multiplicative
- Accounting for the joint (correlation in time) power price distribution by building some heuristic on top of the problem might be interesting

Solving the for an optimal *policy*

- The natural way to find the optimal *policy* - and hence the controller that gives the global optimum – is by using stochastic dynamic programming
- Principle of optimality (Bellmans equation)

$$V_t(x_t) = min_{u \in U} E[\underbrace{L(x_t, u_t, w_t)}_{Immediate\,cost} + \underbrace{V_{t+1}(x_{t+1})}_{cost.to.go}]$$

$$V_N(x_N) = E[\underbrace{\phi(x_N, w_N)}_{Terminal}]$$

- Combining the system equation and Bellmans equation the problem can be solved brute force - discretized state space & terminates in finite time

Stochastic programming con´t

- Pitfalls using brute force
  - Curse of dimensionality
  - Not generally possible to separate predictions and control
- Example of a ”nice” solution
  - Linear system, gaussian uncertainty, quadratic cost (See e.g. ”Intro. to stochastic control theory” Åstrøm)
  - Estimation problem and control problem decoupled
    - Kalman filter (stochastic optimal state estimator)
    - LQ controller (deterministic optimal controller)

Electric car problem - the dice example by Bjarke (Actua)

- Electricity price at time *t* is given by outcome of random variable P_t (fair dice with 6 sides)
- We need to charge the car within the horizon *N*. The car has c_0 units when it start. Full charge is c_L units
- The car can be charged one unit (u=1) or zero units (u=0) in a given sample time.

$$V_0(c_0) = min_{u_{0,} u_{1,}.., u_{N-1}} E[J = \sum_{t=0}^{N-1} u_t P_t + \alpha (c_L - c_N)^2]$$

$$c_{t+1} = c_t + u_t$$

$$c_t \in (0,1,2,\ldots, c_L), u_t \in (0,1)$$

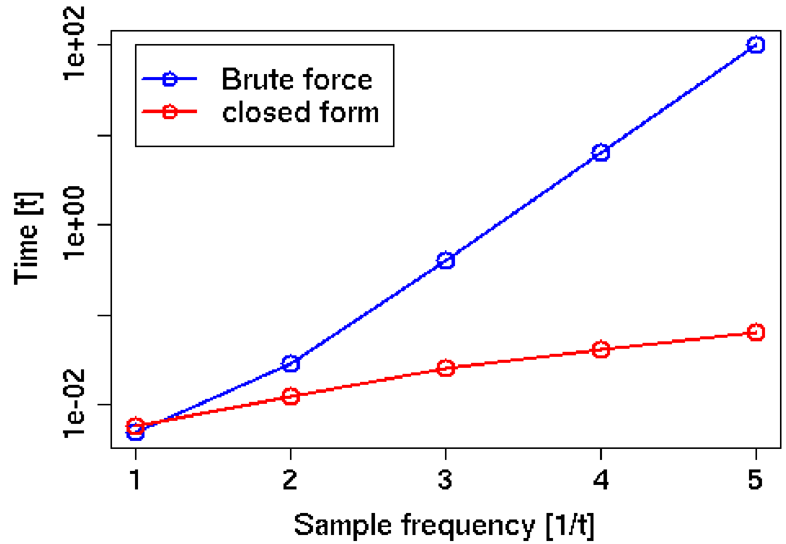
Example con´t

- ”Closed form” solution for α=∞ (i.e.fully charged at *t*=*N*)

$$u(c)=1\,if\;\; p<R_{(c_L-c),N-1}-R_{(c_L-c-1),N-1}$$

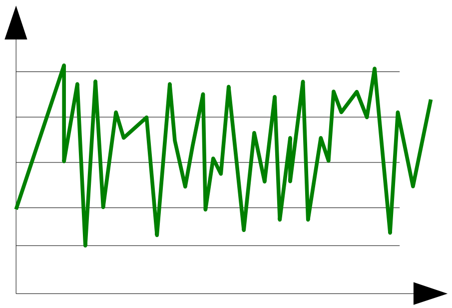
$$R_{i,j}=\frac{1}{6}\sum_{k=1}^{6} min(k+R_{i-1,j-1},R_{i,j-1}) \qquad R_{1,1}=3.5$$

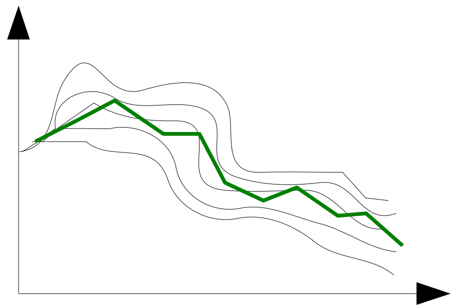
- Numerical example
  - 4 hours in plug
  - Needs 2 hours



Example con´t

- The dice example clearly illustrates that more is needed than the conditional expectation.
- The dice example is however not realistic. Uniform distribution between price limits & no time correlation





- Implications of using receding horizon solution wanish as we move further to the ”right”

Example con´t

- If time allows

Conclusion

- The electric car example shows the need for including the price distribution in the optimization problem. The stochastic programming problem is tractable with the simplified (dice) setup. Accounting for correlated prices might call for heuristics.
- The house heating example is not globally optimal in a math. sence. However, given the complications of solving for a policy and no finite terminal time it is arguably the most practical approach. Dice example shows ”worst case” - real life situation has more structure.